**BCS THEORY**

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BCS theory or Bardeen-Cooper-Schrieffer theory (named after scientists John Bardeen, Leon Cooper and John Robert Schrieffer) is the first microscopic theory of superconductivity since Heike Kamerlingh Onnes’s discovery in 1911. BCS theory describes superconductivity as a microscopic effect caused by a condensation of Cooper pairs. The theory is also used in Nuclear Physics to describe the pairing interaction between nucleons in an atomic nucleus.

This theory was proposed by Bardeen, Cooper and Schrieffer in 1957; they received the Nobel Prize in Physics for this theory in 1972. Rapid progress in the understanding of superconductivity gained momentum in mid-1950s. It began with the 1948 paper, ‘On the Problem of the Molecular theory of Superconductivity’ where Fritz London proposed that the phenomenological London equations may be consequences of the coherence of a quantum state. In 1953, Brian Pippard Motivated by the penetration experiments, proposed that this would modify the London equations via a new scale parameter called the ‘coherence length’. John Bardeen then argued in the 1955 paper ‘Theory of the Meissner Effect in Superconductors’, that such a modification naturally occurs in a theory with an energy gap. The key ingredient was Leon Cooper’s calculation of the bound states of electrons subject to an attractive force in his 1956 paper ‘Bound Electron pairs in a Degenerate Fermi Gas’. Again in 1957, Bardeen and Cooper assembled these ingredients and constructed such a theory – the BCS theory with Robert Schrieffer. The theory was first published in April 1957 in the letter ‘Microscopic theory of superconductivity’. The demonstration that the phase transition is second order, that it reproduces the Meissner Effect and the calculations of the specific heats and penetration depths, appeared in the article in December 1957 – ‘Theory of Superconductivity’. In 1986, high-temperature superconductivity was discovered in La-Ba-Cu-O, at temperatures upto 30 Kelvin. Following experiments determined more materials with transition temperatures upto 130 Kelvin, considerably above the previous limit of about 30 Kelvin. It is believed that BCS theory alone cannot explain this phenomenon and that other effects are in play. These effects are still not yet fully understood – it is possible that they even control superconductivity at low temperatures for some materials.

At sufficiently low temperatures, electrons near the Fermi surface become unstable against the formation of Cooper-pairs. Cooper showed such binding will occur in the presence of an attractive potential, no matter how weak. In conventional superconductors, an attraction is generally attributed to an electron-lattice interaction. The BCS theory however requires only that the potential be attractive, regardless of its origin. In the BCS framework, superconductivity is a macroscopic effect which results from the condensation of Cooper pairs. These have some Bosonic properties and bosons at sufficiently low temperature forming a large Bose-Einstein condensate. Superconductivity was simultaneously explained by Nikolay Bogolyubov, by means of the ‘Bogoliluv transformations’. In many superconductors the attractive interaction between electrons (necessary for pairing) is brought about indirectly by the interaction between the electrons and the vibrating crystal lattice (the phonons).

BCS theory starts from the assumption that there is some attraction between electrons which can overcome the Coulomb repulsion. In most materials (in low-temperature superconductors), this attraction is brought about indirectly by the coupling of electrons to the crystal lattice. However, the results of BCS theory do not depend on the origin of the attractive interaction. For instance, Cooper pairs have been observed in the ultra-cold gases of Fermions, where a homogeneous magnetic field has been turned to their Feshbach resonance. The original results of BCS theory described s-wave superconducting state, which is the rule among low-temperature superconductors but is not realized in many unconventional superconductors such as the d-wave high-temperature superconductors. extensions of BCS theory exist to describe these other cases, although they are insufficient to completely describe the observed features of high-temperature superconductivity.

BCS theory is able to give an approximation for the quantum-mechanical many-body state of the system of attractively interacting electrons inside the metal. This state is now known as the BCS state. In the normal state of a metal, electrons move independently, whereas in the BCS state, they are bound into Cooper pairs by the attractive interaction. The BCS formalism is based on the reduced potential for the attraction of electrons. Within this potential a variational ansatz for the wave function is proposed. This ansatz was later shown to be exact in the dense limit of pairs. The continuous crossover between the dilute and the dense regimes of attracting pairs of fermion is still an open problem, which now attracts a lot of attention within the field of ultra-cold gases.

BCS theory derived several important theoretical predictions, that are independent of the details of the interaction, since the quantitative predictions hold for any sufficiently weak attraction between the electrons – the condition is fulfilled for many low temperature superconductors – the so-called weak coupling case. The electrons are bound into Cooper pairs and these pairs are correlated due to the Pauli exclusion principle for the electrons, for which they are constructed. Therefore, in order to break a pair, one has to change the energies of all other pairs. This means that there is an energy gap for single- particle excitation, unlike in the normal metal (where the state of an electron can be changed by adding an arbitrarily small amount of energy). This energy gap is highest at low temperatures but vanishes at the transition temperature when superconductivity ceases to exist. The BCS theory gives an expression that shows hoe the gap grows with the strength of the attractive interaction and the normal phase single particle density of states at the Fermi level. Furthermore, it describes how the density of states is changed on entering the superconducting state, where there are no electronic states anymore at the Fermi level. The energy gap is more directly observed in tunnelling experiments and in reflection of micro waves from superconductors. BCS theory predicts the dependence of the value of energy gap at a temperature over the critical temperature. The ratio between the value of energy gap at zero temperature and the value of superconducting transition temperature, expressed in energy units, takes the universal value (T=0)= 1.764 units, independent of material. Due to the energy gap, the specific heat of the superconductor is uppressed strongly (exponentially) at low temperatures, there being no thermal excitations left. However before reaching the transition temperature, the specific heat of the superconductor becomes even higher than that of the normal conductor (measured immediately above the transition) and the ratio of these two values is found to be universally given by 2:5.

The BCS theory reproduces the isotope effect, which is the experimental observation, that for a given superconducting material, the critical temperature is inversely proportional to the mass of the isotope used in the material. The isotope effect was reported by two groups on 24th March 1950, who discovered it independently working with different mercury isotopes, although a few days before publication, they learned of each other’s result at the ONR conference in Atlanta. The two groups are Emanuel Maxwell and C.A. Reynolds, B. Serin, W.H.Wright and L.B. Nesbitt. The choice of isotope ordinarily has little effect on the electrical properties of a material but does affect the frequency of lattice vibrations. This effect suggests that superconductivity is related to the vibrations of the lattice. This is incorporated into BCS theory, where lattice vibrations yield the binding energy of electrons in a Cooper pair.